Study of the Short Term Stability of Crystal Oscillators

SECOND QUARTERLY REPORT

October 1, 1961 to December 31, 1961

Report Number 6

Contract Number 2

(continuation of Contract Number 1)

638 300

…EW YORK UNIVERSITY

…GE OF ENGINEERING

…Electrical Engineering

…53, N. Y.

STUDY OF SHORT TERM STABILITY OF CRYSTAL OSCILLATORS

J. Rarity
L. Saporta
G. Weiss

Object of Research: The object of the research program is to conduct an investigation of the factors affecting short term frequency stability of crystal oscillators.

NEW YORK UNIVERSITY
COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING

University Heights
New York 53, New York

SECOND QUARTERLY REPORT

October 1, 1961 to December 31, 1961

Report Number 6

Contract No. 2
(Continuation of Contract No. 1)
Contract No. DA-36-039-sc-87450
DA Project No. 3A-99-15-021-02-02

Prepared
for
FREQUENCY CONTROL DIVISION
ELECTRONICS COMPONENTS RESEARCH DEPARTMENT
U.S. ARMY SIGNAL RESEARCH AND DEVELOPMENT LABORATORY
HEXAGON BUILDING
FORT MONMOUTH, NEW JERSEY

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. PURPOSE OF CONTRACT

The purpose of the contract is to expose the underlying phenomena producing short term frequency variations and to advance the art of short term stability measurement techniques. The results of the investigation should lead to the subsequent improvement of the short term stability of high precision crystal oscillators.

II. ABSTRACT

Two definitions of frequency stability are discussed. One is basically the variance of the frequency as determined in an observation time τ. The second definition involves the variance of sample pairs of the first, the samples being separated by some time T. These definitions are shown to be simply related to each other. Since each is derivable from the spectrum of the noise, it is proposed that a convenient way of describing frequency stability is to specify the spectrum of the oscillator. From the spectrum either of the above, as well as other definitions of stability, may be derived to best suit a particular application.

The remainder of the report describes the work done so far on the analysis of oscillator non-linearities as they affect the frequency scatter introduced by noise. The spectrums of the quadrature components of the noise for a simple model of a non-linear oscillator have been derived. Further work is required to access the importance of the non-linearity on the resultant frequency scatter.

III. PUBLICATIONS, LECTURES, REPORTS AND CONFERENCES

Conferences:

Place: USASRDL, Fort Monmouth, New Jersey

Date: 30 November 1961

Participants: Messrs. Layden, Schodowsky, Dr. Hafner, (USASRDL)
Messrs. Rarity, Saporta, Weiss, (NYU)

Subject: Discussion of first quarterly report of subject contract.

Reports:

Monthly Progress Reports

1. October 1, 1961 to October 31, 1961
2. December 1, 1961 to December 31, 1961

Lectures:

Lecture given at New York University, College of Engineering, Department of Electrical Engineering Colloquium, by Lester Saporta on December 19, 1961, entitled, "Basic Problems in Frequency Stability".

IV. FACTUAL DATA

A. Introduction

In the first quarter of this contract a study of the linear addition of various noise spectrums to a fixed sinusoid was made. The effects of the noise on the scatter of the observed frequency of the sinusoid was computed. The present report extends this work in two respects. First the definition of frequency scatter has been somewhat generalized to permit the computation of the scatter among sample pairs of the observed frequency when these samples are separated by a given time. This definition may be more appropriate when a figure of merit is desired in certain timing applications.

The second extension of the analysis involves the evaluation of non-linear effects in the oscillator on the frequency scatter. This analysis is not yet complete but the progress to date is reported on.

B. Alternate Definitions of Frequency Stability

The statistical properties of the phase accumulated in a time τ by a sinusoid added to various types of noise were discussed in the first quarterly report of this contract.

The r.m.s. value of this accumulated phase was determined as a function of the measurement time τ. This led to a definition of frequency stability as the r.m.s. phase accumulated divided by the time τ.

In certain applications it may be desirable to extend this definition. For example the velocity of an object might be determined by comparing the transmitted frequency to that of the received target echo. The signal might be transmitted in bursts of duration τ corresponding to the time variable mentioned above, but the time, T, between the transmission of the burst and the reception of the echo might be considerably longer than τ. In this case the performance of the velocity measuring system depends on the scatter among sample pairs of duration τ and spacing T. The figure of merit appropriate to the evaluation of an oscillator for this purpose thus depends on both τ and T. This involves a somewhat more complicated definition of frequency stability than the previously used definition which depends only on τ.

The definition of frequency stability extended in this sense is easily derived from the simpler definition discussed in

the previous report. This will be developed in the following paragraphs.

In the first quarterly report it was shown that the effect of adding noise to a fixed sinusoid results, for large signal-to-noise ratios, is an effective modulation of the phase of the sinusoid. In the figure below, the quantity $\theta(t)$ represents a typical record of the deviation from the nominal phase that would accrue in the absence of noise. From this diagram, it may be seen that

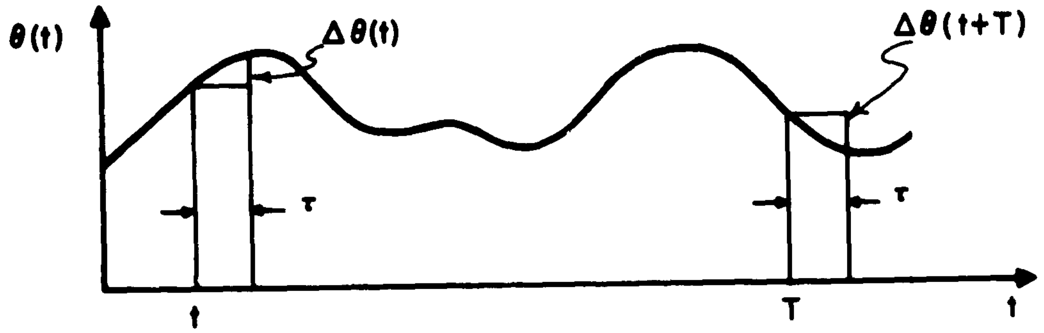


Figure 1 - Sketch of a representative record of the composite phase angle $\theta(t)$

at time t, the change in phase over a short time interval τ is $\Delta\theta(t)$. At time $t + T$, the change in phase over the same short time interval τ is $\Delta\theta(t + T)$. The change in phase (or accumulated phase) may be written in terms of the absolute phase in the following way:

$$\Delta\theta(t) = \theta(t + \tau) - \theta(t) \quad , \tag{B-1}$$

and:

$$\Delta\theta(t + T) = \theta(t + T + \tau) - \theta(t + T) \quad . \tag{B-2}$$

Corresponding to the change in phase as given above, a frequency

deviation from the nominal frequency is expressible in accordance with the definition of frequency used in this development. Thus,

$$\omega(t) = \frac{\Delta\theta(t)}{\tau} \quad , \tag{B-3}$$

and:

$$\omega(t + T) = \frac{\Delta\theta(t + T)}{\tau} \quad . \tag{B-4}$$

The difference in frequency $\Delta\omega(T, \tau)$ measured at the beginning and at the end of the time interval T, may be written as:

$$\Delta\omega(T, \tau) = \omega(t + T) - \omega(T) = \frac{1}{\tau}[\Delta\theta(t + T) - \Delta\theta(t)] \quad . \tag{B-5}$$

Or, in terms of the absolute phase,

$$\Delta\omega(T, \tau) = \frac{1}{\tau}(\theta_4 - \theta_3 - \theta_2 + \theta_1) \quad , \tag{B-6}$$

where, for convenience in writing the absolute phase, the following notation has been used:

$$\theta_4 = \theta(t + T + \tau) \quad ,$$

$$\theta_3 = \theta(t + T) \quad ,$$

$$\theta_2 = \theta(t + \tau) \quad ,$$

$$\theta_1 = \theta(t) \quad . \tag{B-7}$$

The mean square value of $\Delta\omega(T, \tau)$ is therefore:

$$\sigma_\omega^2(T, \tau) = \overline{\Delta^2\omega(T, \tau)} = \frac{1}{\tau^2} \overline{(\theta_4 - \theta_3 - \theta_2 + \theta_1)^2} \tag{B-8}$$

Upon expanding the right-hand side of (B-8), and noting that $\overline{\theta_i\theta_j} = \overline{\theta_j\theta_i}$, the following expression is obtained:

$$\sigma_\omega^2(T, \tau) = \tag{B-9}$$

$$\frac{1}{\tau^2}(\overline{\theta_1^2} + \overline{\theta_2^2} + \overline{\theta_3^2} + \overline{\theta_4^2} - 2\overline{\theta_1\theta_2} - 2\overline{\theta_1\theta_3} + 2\overline{\theta_1\theta_4} + 2\overline{\theta_2\theta_3} - 2\overline{\theta_2\theta_4} - 2\overline{\theta_3\theta_4}) \; .$$

The quantities $\overline{\theta_i\theta_j}$ may be written in terms of the corresponding auto-correlation function in the following manner:

$$\overline{\theta_1^2} = \overline{\theta_2^2} = \overline{\theta_3^2} = \overline{\theta_4^2} = R(0) \quad , \tag{B-10}$$

$$\overline{\theta_1\theta_2} = \overline{\theta(t)\ \theta(t + \tau)} = R(\tau) \quad , \tag{B-11}$$

$$\overline{\theta_1\theta_3} = \overline{\theta(t)\ \theta(t + T)} = R(T) \quad , \tag{B-12}$$

$$\overline{\theta_1\theta_4} = \overline{\theta(t)\ \theta(t + T + \tau)} = R(T + \tau) \quad , \tag{B-13}$$

$$\overline{\theta_2\theta_3} = \overline{\theta(t + \tau)\ \theta(t + T)} = R(T - \tau) \quad , \tag{B-14}$$

$$\overline{\theta_2\theta_4} = \overline{\theta(t + \tau)\ \theta(t + T + \tau)} = R(T) \quad , \tag{B-15}$$

$$\overline{\theta_3\theta_4} = \overline{\theta(t + T)\ \theta(t + T + \tau)} = R(\tau) \quad . \tag{B-16}$$

Substitution of expressions (B-10) through (B-16) into (B-9) yields:

$$\sigma_\omega^2(T,\ \tau) = \frac{1}{\tau^2}[4R(0) - 4R(\tau) - 4R(T) + 2R(T + \tau) + 2R(T - \tau)] \quad . \tag{B-17}$$

Rearranging expression (B-17) results in:

$$\sigma_\omega^2(T,\ \tau) = \frac{1}{\tau^2}\Big\{4[R(0) - R(\tau)] + 4[R(0) - R(T)] - 2[R(0) - R(T + \tau)] - 2[R(0) - R(T - \tau)]\Big\} \quad . \tag{B-18}$$

In the first quarterly report of the contract, expressions for the mean square value of the accumulated phase, for a single sample of duration τ seconds, were derived for a sinusoid perturbed by various types of random noise. This result may be written in terms of some arbitrary correlation time delay, ξ , in the following way:

$$\sigma_{\Delta\theta}^2(\xi) = 2[R(0) - R(\xi)] \quad , \tag{B-19}$$

and may be used to evaluate $\sigma_\omega^2(T,\ \tau)$ given in expression (B-18). The procedure for carrying out this evaluation is to determine the value of $\sigma_{\Delta\theta}^2(\xi)$ for ξ equal to τ, T and T $\pm$ τ and substitute these values in (B-18). Consequently expression (B-18) may be written as:

$$\sigma_\omega^2(T,\ \tau) = \frac{1}{\tau^2}\left[2\,\sigma_{\Delta\theta}^2(\xi)\Big|_{\xi=\tau} + 2\,\sigma_{\Delta\theta}^2(\xi)\Big|_{\xi=T} - \sigma_{\Delta\theta}^2(\xi)\Big|_{\xi=T+\tau} - \sigma_{\Delta\theta}^2(\xi)\Big|_{\xi=T-\tau}\right] \quad . \tag{B-20}$$

To illustrate the evaluation of $\sigma_{\omega}^{2}(T, \tau)$, consider the large signal-to-noise ratio approximation of the linear addition of a sinusoid and random noise. The result for $\sigma_{\Delta\theta}^{2}(\xi)$ when the spectrum of the random noise is shaped by a Gaussian filter is sketched in Figure 2.

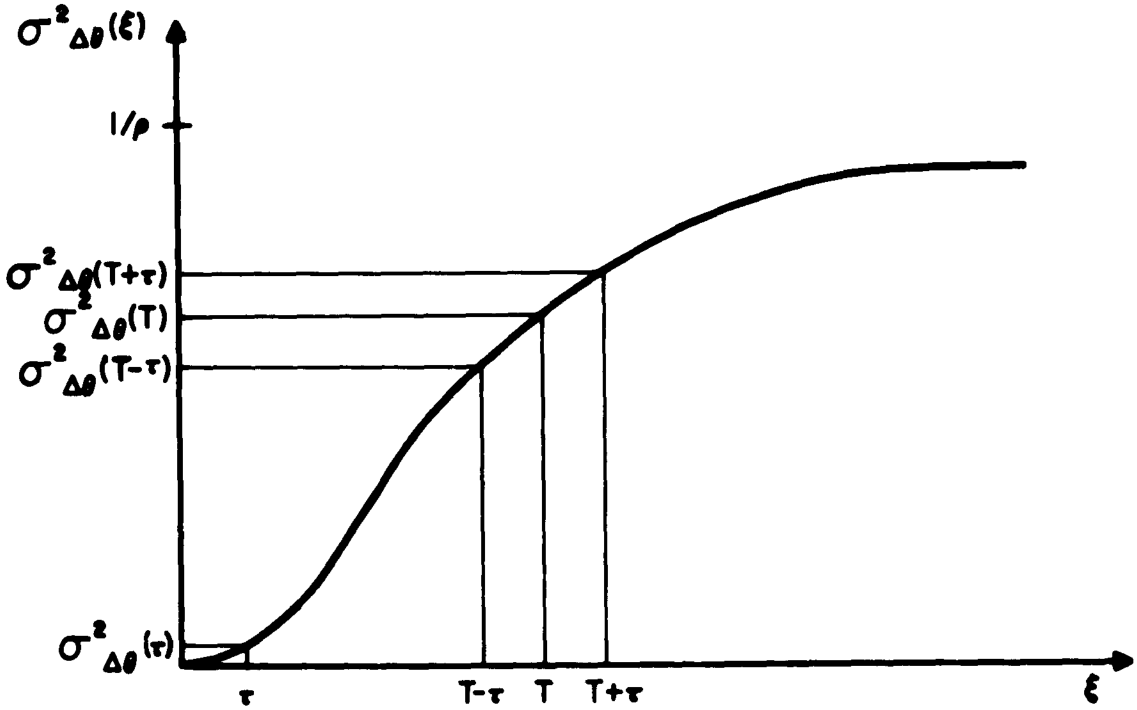


Figure 2 - Sketch of the variance of the accumulated phase for noise shaped by Gaussian filter

From the above, it is seen how the variance in the accumulated phase $\Delta\theta$ is determined at the four delay times τ, T, and $T + \tau$. Knowledge of the mean square value R(0) and the auto-correlation function $R(\xi)$ of the random accumulated phase is sufficient to determine $\sigma_{\Delta\theta}^{2}(\xi)$.

From the above development, it has been shown that an expression for the scatter in frequency based on sample pairs is derivable in terms of the scatter based on a single sample as developed in the first quarterly report. In view of this, the scatter may be determined from either definition from a knowledge of the auto-correlation of the noise which in turn is related to the power spectrum of the noise.

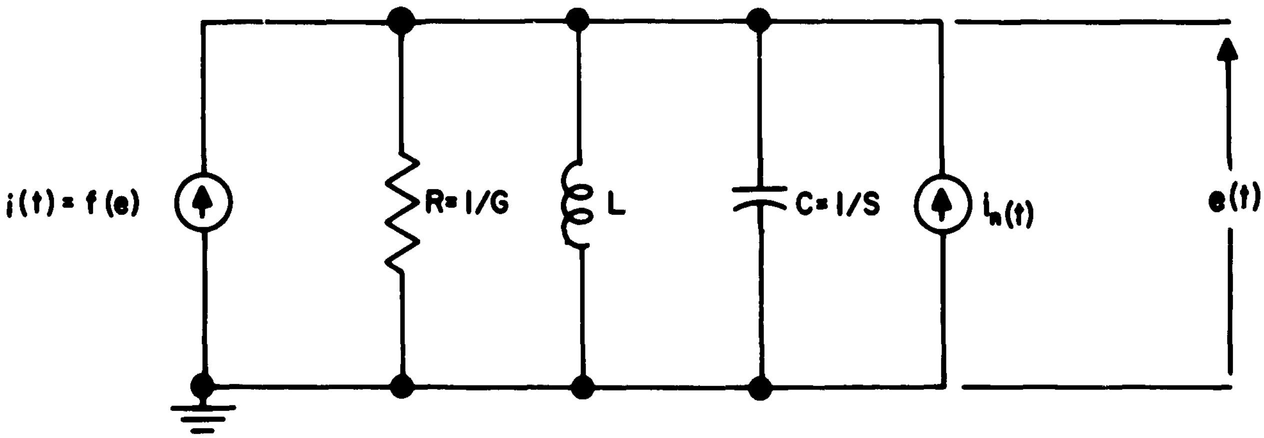
C. Non-Linear Effects on Frequency-Scatter

This section of the report is devoted to a discussion of the effects of noise in a non-linear oscillator. The previous report considered the case of the linear addition of various noise spectrums to a fixed sinusoid. The scatter in the accumulated phase and frequency due to the noise was computed as a function of the observation or measurement time. Essentially the same type of analysis is required in the non-linear case, but the determination of the effective noise spectrum is a somewhat more difficult problem than is the case for a linear analysis.

The following analysis represents the progress made in this problem to date. Further work will be required to permit the evaluation of the scatter in terms of the frequency spectrum which will be derived.

The method of approach was suggested by an unpublished paper of Dr. Eric Hafner of the United States Army Signal Research and Development Laboratories.

The circuit shown in Figure 3 represents the tank circuit of a typical oscillator. The current generator $i(t)$ provides for the feedback due to the presence of the active element necessary to sustain oscillations. In general this element will be non-linear and this is taken into account by making $i(t)$ a function of the instantaneous voltage across the tank.



<u>Figure 3</u> - Model of Non-Linear Oscillator

The generator $i_n(t)$ represents the noise produced in the resistor, R. For the purposes of this analysis $i_n(t)$ will be assumed to have a white frequency spectrum. For this network we have

$$C\dot{e} + \frac{e}{R} + \frac{1}{L}\int e\,dt = i(t) + i_n(t) \qquad , \qquad \text{(C-1)}$$

$$\dot{e} + GSe + \omega_o^2 \int e\,dt - Sf(e) = Si_n(t) \qquad , \qquad \text{(C-2)}$$

where $\qquad \omega_o^2 = \frac{1}{LC} \qquad .$

If the steady state solution of the homogenous equation obtained by removing the noise source is designated $e_o(t)$, equation (C-2) becomes

$$\dot{e}_o + GSe_o + \omega_o^2 \int e_o dt - Sf(e_o) = 0 \qquad . \qquad \text{(C-3)}$$

Upon differentiation with respect to time this becomes

$$\ddot{e}_o + GS\dot{e}_o + \omega_o^2 e_o - Sf'(e_o)\,\dot{e}_o = 0 \quad , \tag{C-4}$$

where the prime refers to differentiation with respect to e.

In the weakly non-linear case of interest here, the steady state solution will be approximately given by:

$$e_o = A\cos\omega_o t \quad ; \quad \dot{e}_o = -A\omega_o \sin\omega_o t \quad ; \quad \ddot{e}_o = -A\omega_o^2 \cos\omega_o t \quad . \tag{C-5}$$

Substitution of these results into (C-4) yields

$$G\dot{e}_o - f'(e_o)\,\dot{e}_o = 0 \quad . \tag{C-6}$$

Since $e_o(t) = A\cos\omega_o t$ is an even periodic function of time with fundamental ω_o, $f'(e_o)$ may be expanded in a Fourier Cosine Series:

$$f'(e_o) = F_o + \sum_{n=1}^{\infty} 2F_n \cos n\omega_o t \quad , \tag{C-7}$$

where $$F_n = \frac{\omega_o}{2\pi}\int_{-\pi/\omega_o}^{\pi/\omega_o} f'(e_o)\, e^{-jn\omega_o t}\, dt \quad .$$

The differential equation (C-6) must in the steady state be satisfied by each harmonic of the solution. Thus collecting the first harmonic terms after substituting (C-5) and (C-7) in (C-6):

$$G\cdot(-A\omega_o \sin\omega_o t) - F_o\cdot(-A\omega_o \sin\omega_o t) + F_2\cdot(-A\omega_o \sin\omega_o t) = 0$$

or $$G - F_o + F_2 = 0 \tag{C-8}$$

Equation (C-8) establishes the equilibrium conditions of the network without noise. When a particular non-linear function $f(e)$ is specified, equation (C-8) permits the evaluation of the amplitude of the oscillations.

Assuming that the steady state oscillations have been established, the noise generator may be reconnected. The resultant solution for the output voltage may be written as the sum of the solution without noise, $e_o(t)$ and some increment $u(t)$. Thus

$$e(t) = e_o(t) + u(t) \quad . \tag{C-9}$$

Substituting (C-9) in equation (C-2) results in

$$\dot{e}_o + GSe_o + \omega_o^2 \int e_o dt - Sf(e_o+u) + \dot{u} + GSu + \omega_o^2 \int udt = Si_n \ . \tag{C-10}$$

For u very small

$$- Sf(e_o+u) \approx - Sf(e_o) - Sf'(e_o)u \quad . \tag{C-11}$$

With this approximation, and in view of equation (C-3), equation (C-11) reduces to

$$\dot{u} + S[G - f'(e_o)]u + \omega_o^2 \int udt = Si_n \quad , \tag{C-12}$$

or

$$\dot{u} + S[G - F_o - \sum_{n=1}^{\infty} 2F_n \cos n\omega_o t]\ u + \omega_o^2 \int udt = Si_n \quad . \tag{C-13}$$

Since the tuned circuit is narrow-band, u can have important frequency components only in the vicinity of ω_o. Neglecting all components far removed from ω_o, and using the equilibrium relation of equation (C-8), equation (C-13) becomes:

$$\dot{u} - SF_2(1 + 2 \cos 2\omega_o t)\, u + \omega_o^2 \int u dt = Si_n \quad . \tag{C-14}$$

Note that equation (C-14) is a linear equation in u with time varying coefficients. The linear property permits the use of superposition in obtaining a solution.

Assume a solution for u(t) in the form

$$u(t) = x \cos \omega_o t - y \sin \omega_o t \tag{C-15}$$

Then

$$\dot{u}(t) = (\dot{x}-\omega_o y) \cos \omega_o t - (\dot{y}+\omega_o x) \sin \omega_o t \tag{C-16}$$

The integral of u with respect to t is most conveniently obtained by assuming the form:

$$\int u dt = a \cos \omega_o t - b \sin \omega_o t \quad . \tag{C-17}$$

Differentiating (C-17) provides:

$$u = (\dot{a}-\omega_o b) \cos \omega_o t - (\dot{b}+\omega_o a) \sin \omega_o t \quad . \tag{C-18}$$

Comparing (C-18) with (C-15) results in

$$x = \dot{a} - \omega_o b \quad ; \quad y = \dot{b} + \omega_o a \quad . \tag{C-19}$$

Denoting by a capital letter the Fourier transform of the corresponding lower case time function (C-19) becomes

$$X = j\omega A - \omega_o B \quad ; \quad Y = j\omega B + \omega_o A \quad . \tag{C-20}$$

Equations (C-20) may be solved simultaneously for A and B:

$$A = \frac{j\omega X + \omega_o Y}{\omega_o^2 - \omega^2} \quad ; \quad B = \frac{j\omega Y - \omega_o X}{\omega_o^2 - \omega^2} \quad . \tag{C-21}$$

The time functions a(t) and b(t) may be recovered by taking the inverse transforms of $A(\omega)$ and $B(\omega)$, i.e., $a(t) = F^{-1}[A(\omega)]$, etc. However, since the remainder of the analysis will be carried out in the frequency domain it is not necessary to explicitly perform this step.

Substituting (C-15), (C-16), and (C-17) into (C-14), collecting $\cos \omega_o t$ and $\sin \omega_o t$ terms, and eliminating third harmonic components results in

$$\begin{aligned} &(\dot{x} - \omega_o y - 2SF_2 x + \omega_o^2 a) \cos \omega_o t \\ &\quad - (\dot{y} + \omega_o x + \omega_o^2 b) \sin \omega_o t = Si_c \cos \omega_o t - Si_s \sin \omega_o t \quad , \end{aligned} \tag{C-22}$$

where the noise generator $i_n(t)$ has been written as the sum of $\cos \omega_0 t$ and $\sin \omega_0 t$ components.

Separately equating $\cos \omega_0 t$ and $\sin \omega_0 t$ components on each side of the equation and taking Fourier transforms, provides the pair of simultaneous equations:

$$\left(j\omega - 2SF_2 + \frac{\omega_0^2\, j\omega}{\omega_0^2 - \omega^2}\right) X + \left(-1 + \frac{\omega_0^2}{\omega_0^2 - \omega^2}\right)\omega_0 Y = SI_c$$

$$\left(1 - \frac{\omega_0^2}{\omega_0^2 - \omega^2}\right)\omega_0 X + \left(j\omega + \frac{\omega_0^2\, j\omega}{\omega_0^2 - \omega^2}\right) Y = SI_s \qquad \text{(C-23)}$$

These two equations may be solved for $X(\omega)$ and $Y(\omega)$ in terms of $I_c(\omega)$ and $I_s(\omega)$. The resultant expressions represent the Fourier spectrums of the cosine and sine components of the required voltage perturbation. The techniques for evaluating the mean square values of $x(t)$ and $y(t)$, and the relation of these quantities to the amplitude and phase of the total output voltage are similar to those outlined in the previous report. The actual integrations involved, however, while straightforward in principle are unwieldy, and therefore the completion of this problem will require some simplification of the equations or perhaps a resort to graphical integration. The completion of this phase of the investigation will therefore be reserved for the next quarterly interval.

V. CONCLUSIONS

In discussing short term frequency stability, various definitions may be appropriate to different physical situations. The relationship between two definitions which appear to have frequent application was explored in the first section of this report. It was shown that the statistics appropriate to one definition are easily derived from those of the other. This in turn reflects the fact that the statistics referring to either definition of short term stability are related to the power spectrum of the signal. In view of this it seems appropriate to characterize an oscillator by specifying its spectrum rather than selecting any particular definition of short term stability as a standard. This has the advantage of permitting the user of the information the choice of definition most appropriate to his own application. If this approach is used, the frequency scatter measurements made with the multiplier-heterodyne system may be regarded as a method of determining the auto-correlation function of the oscillator frequency which in turn may be used to obtain the spectrum. In effect the multiplier-heterodyne system plays the role of a narrow band spectrum analyzer using time domain filtering.

The second half of this report was devoted to a study of non-linear effects in oscillators on the frequency scatter introduced by noise. A model comparable to the linear models,

which were studied in the previous report, was analyzed following a technique suggested by Dr. Eric Hafner, and related to the van der Pol approach. When completed, the analysis should indicate whether a significant difference is introduced by the non-linearity, or whether the simpler linear analysis is adequate for most purposes. The study has been carried to a point where the spectra of the quadrature components of the output are available. Some further work is required to compare these results with the previously derived linear results.

VI. PROGRAM FOR THE NEXT INTERVAL

An important item in the work scheduled for the next quarter is the completion of the study of the non-linear effects described in Section C. In particular it is desirable to be able to indicate whether non-linearities play a relatively important or unimportant role in determining the frequency-scatter due to noise.

A second item on the schedule calls for an investigation of frequency determining techniques using a feedback system to phase lock the unknown oscillator to a standard.

Some experimental work on the multiplier-heterodyne system is also contemplated. A known frequency perturbation will be introduced into an oscillator and the ability of the system to reveal the nature of the perturbation will be studied.

VII. IDENTIFICATION OF KEY TECHNICAL PERSONNEL

The technical personnel employed on the subject contract and the total number of hours expended by each during the contract period are as follows:

		Hours
J. Rarity	Research Scientist	290
L. Saporta	Senior Research Scientist	228
G. Weiss	Research Scientist	305

VIII. BIBLIOGRAPHY

1. Rarity, J., Saporta, L., and Weiss, G. "Study of the Short Term Stability of Crystal Oscillators." <u>1st Quarterly Report</u>, (July 1, 1960 to September 30, 1960), Contract DA-36-039-sc-85376.

2. Rarity, J., Saporta, L., and Weiss, G. "Study of the Short Term Stability of Crystal Oscillators." <u>2nd Quarterly Report</u>, (October 1, 1960 to December 31, 1960), Contract DA-36-039-sc-85376.

3. Rarity, J., Saporta, L., and Weiss, G. "Study of the Short Term Stability of Crystal Oscillators." <u>3rd Quarterly Report</u>, (January 1, 1961 to March 31, 1961), Contract DA-36-039-sc-85376.

4. Rarity, J., Saporta, L., and Weiss, G. "Study of the Short Term Stability of Crystal Oscillators." <u>Final Report</u>, (July 1, 1960 to June 30, 1961), Contract DA-36-039-sc-85376.

5. Rarity, J., Saporta, L., and Weiss, G. "Study of the Short Term Stability of Crystal Oscillators." <u>1st Quarterly Report</u>, (July 1, 1961 to September 30, 1961), Contract DA-36-039-sc-87450.

DISTRIBUTION LIST

	Copy no.
OASD (R and E), Room 3E1065 ATTN: Technical Library The Pentagon Washington 25, D. C.	1
Chief of Research and Development OCS, Department of the Army Washington 25, D. C.	2
Chief Signal Officer ATTN: SIGRD-7 Department of the Army Washington 25, D. C.	3
Director U. S. Naval Research Laboratory ATTN: Code 2027 Washington 25, D. C.	4
Commanding Officer and Director U. S. Navy Electronics Laboratory San Diego 52, California	5
Chief, Bureau of Ships, Code 690B Department of the Navy ATTN: Mr. R. B. McDowell Washington 25, D. C.	6
Commander Wright Air Development Division ATTN: WCOSI-3 Wright-Patterson AFB, Ohio	7-8
Commander Wright Air Development Division ATTN: WCLNE, Mr. C. Friend Wright-Patterson AFB, Ohio	9
Commander Aeronautical Systems Division ATTN: ASNPVE-2, Mr. E. Borgelt Wright-Patterson AFB, Ohio	10
Commander Rome Air Development Center ATTN: RCSSLD Air Research and Development Command Griffiss AFB, New York	11
Secretariat Advisory Group on Electronic Parts, R and E Moore School Building 200 So. 33rd Street Philadelphia 4, Pennsylvania	12-15
Chief U. S. Army Security Agency Arlington Hall Station Arlington 12, Virginia	16-17
Deputy President U. S. Army Security Agency Board Arlington Hall Station Arlington 12, Virginia	18
Commander Armed Services Technical Information Agency ATTN: TIPDR Arlington Hall Station Arlington 12, Virginia	19-28
Commanding Officer U. S. Army Signal Material Support Agency ATTN: Adjutant Fort Monmouth, New Jersey	29
National Bureau of Standards Boulder Laboratories Boulder, Colorado ATTN: Mr. W. D. George	30
Commander Air Force Cambridge Research Center ATTN: CRXTR L. G. Hanscom Field Bedford, Massachusetts	31
Dr. [illegible] E. [illegible] McMurry College Abilene, Texas	32
Mr. Roger A. Sykes Bell Telephone Laboratories, Inc. Merrimack Valley No. Andover, Massachusetts	33
Hallamore Electronics Division 714 No. Brookhurst Street Anaheim, California	34
Manson Laboratories, Inc. 375 Fairchild Avenue Stamford, Connecticut ATTN: Mr. J. Guttman	35
Motorola, Inc. 8201 East McDowell Scottsdale, Arizona ATTN: Mr. John M. Ross	36
National Bureau of Standards Boulder Laboratories Boulder, Colorado ATTN: Mr. William R. Atkinson, Radio Standards Laboratory	37
National Bureau of Standards Boulder Laboratories Boulder, Colorado ATTN: Mrs. V. S. Barker, Library	38
Radio Corporation of America Building 10-8 Camden 2, New Jersey ATTN: Mr. M. B. Heracher	39
Sylvania Electric Products, Inc. 1100 Wehrle Drive Buffalo 21, New York ATTN: Miss Marion Veath Librarian	40
McDonnell Aircraft Corp. Post Office Box 516 St. Louis 66, Missouri ATTN: Engineering Library Dept. 218	41
Commanding Officer Diamond Ordnance Fuze Laboratories ATTN: Library Washington 25, D. C.	42
Bendix Systems Division 3300 Plymouth Road Ann Arbor, Michigan ATTN: Mr. Vern Hutton	43
Frontier Electronics Co. 4600 Memphis Avenue Cleveland 9, Ohio	44
General Electric Co. Advanced Guidance Engineering 300 South Geddes Street Syracuse, New York ATTN: Mr. Calvin R. Woods, Rm. 111	45
Polartron, Inc. 70 S. Seventh Avenue Long Branch, New Jersey ATTN: Mr. H. D. Tanzman	46
Dr. D. E. Newell Pioneer Central Division of Bendix Corp. Davenport, Iowa	47
Commanding Officer U. S. Army Signal Research and Development Laboratory ATTN: SIGRA/SL-LNF Fort Monmouth, New Jersey	48
ATTN: SIGRA/SL-LNE	49
ATTN: SIGRA/SL-LNR	50
ATTN: SIGRA/SL-DR	51
ATTN: SIGRA/SL-ADT	52
ATTN: SIGRA/SL-PF, Tech Staff, (Record Copy)	53
ATTN: SIGRA/SL-TN	54-56
ATTN: SIGRA/SL-PF, Dr. E. A. Gerber	57
ATTN: SIGRA/SL-PF, Mr. G. Goldenberg	58-71, 75
ATTN: SIGRA/SL-PFP, Miss M. Herberg	72-74

This contract is supervised by the Solid State and Frequency Control Division, Electronic Components Department, USASRDL, Fort Monmouth, New Jersey. For further technical information contact Mr. S. Schodowski, Project Engineer, Telephone 535-2602.

AD ______________ Accession No. ______________

STUDY OF SHORT TERM STABILITY OF CRYSTAL OSCILLATORS

J. Barity, L. Saporta, G. Weiss

Report No. 6, Second Quarterly Report, October 1, 1961 to December 31, 1961
23 pp. Illus, Signal Corps Contract No. DA-36-039-sc-87450, DA No. 3A-99-15-021-02-02

Two definitions of frequency stability are discussed. One is basically the variance of the frequency as determined in an observation time τ. The second definition involves the variance of sample pairs of the first, the samples being separated by some time T. These definitions are shown to be simply related to each other. Since each is derivable from the spectrum of the noise, it is proposed that a convenient way of describing frequency stability is to specify the spectrum of the oscillator. From the spectrum either of the above, as well as other definitions of stability, may be derived to best suit a particular application.

The remainder of the report describes the work done so far on the analysis of oscillator non-linearities as they affect the frequency scatter introduced by noise. The spectrums of the quadrature components of the noise for a simple model of a non-linear oscillator have been derived. Further work is required to access the importance of the non-linearity on the resultant frequency scatter.

1. Short Term Stability
2. Signal Corps Contract DA-36-039-sc-87450

AD ______________ Accession No. ______________

STUDY OF SHORT TERM STABILITY OF CRYSTAL OSCILLATORS

J. Barity, L. Saporta, G. Weiss

Report No. 6, Second Quarterly Report, October 1, 1961 to December 31, 1961
23 pp. Illus, Signal Corps Contract No. DA-36-039-sc-87450, DA No. 3A-99-15-021-02-02

Two definitions of frequency stability are discussed. One is basically the variance of the frequency as determined in an observation time τ. The second definition involves the variance of sample pairs of the first, the samples being separated by some time T. These definitions are shown to be simply related to each other. Since each is derivable from the spectrum of the noise, it is proposed that a convenient way of describing frequency stability is to specify the spectrum of the oscillator. From the spectrum either of the above, as well as other definitions of stability, may be derived to best suit a particular application.

The remainder of the report describes the work done so far on the analysis of oscillator non-linearities as they affect the frequency scatter introduced by noise. The spectrums of the quadrature components of the noise for a simple model of a non-linear oscillator have been derived. Further work is required to access the importance of the non-linearity on the resultant frequency scatter

1. Short Term Stability
2. Signal Corps Contract DA-36-039-sc-87450

AD ______________ Accession No. ______________

STUDY OF SHORT TERM STABILITY OF CRYSTAL OSCILLATORS

J. Barity, L. Saporta, G. Weiss

Report No. 6, Second Quarterly Report, October 1, 1961 to December 31, 1961
23 pp. Illus, Signal Corps Contract No. DA-36-039-sc-87450, DA No. 3A-99-15-021-02-02

Two definitions of frequency stability are discussed. One is basically the variance of the frequency as determined in an observation time τ. The second definition involves the variance of sample pairs of the first, the samples being separated by some time T. These definitions are shown to be simply related to each other. Since each is derivable from the spectrum of the noise, it is proposed that a convenient way of describing frequency stability is to specify the spectrum of the oscillator. From the spectrum either of the above, as well as other definitions of stability, may be derived to best suit a particular application.

The remainder of the report describes the work done so far on the analysis of oscillator non-linearities as they affect the frequency scatter introduced by noise. The spectrums of the quadrature components of the noise for a simple model of a non-linear oscillator have been derived. Further work is required to access the importance of the non-linearity on the resultant frequency scatter.

1. Short Term Stability
2. Signal Corps Contract DA-36-039-sc-87450

AD ______________ Accession No. ______________

STUDY OF SHORT TERM STABILITY OF CRYSTAL OSCILLATORS

J. Barity, L. Saporta, G. Weiss

Report No. 6, Second Quarterly Report, October 1, 1961 to December 31, 1961
23 pp. Illus, Signal Corps Contract No. DA-36-039-sc-87450, DA No. 3A-99-15-021-02-02

Two definitions of frequency stability are discussed. One is basically the variance of the frequency as determined in an observation time τ. The second definition involves the variance of sample pairs of the first, the samples being separated by some time T. These definitions are shown to be simply related to each other. Since each is derivable from the spectrum of the noise, it is proposed that a convenient way of describing frequency stability is to specify the spectrum of the oscillator. From the spectrum either of the above, as well as other definitions of stability, may be derived to best suit a particular application.

The remainder of the report describes the work done so far on the analysis of oscillator non-linearities as they affect the frequency scatter introduced by noise. The spectrums of the quadrature components of the noise for a simple model of a non-linear oscillator have been derived. Further work is required to access the importance of the non-linearity on the resultant frequency scatter.

1. Short Term Stability
2. Signal Corps Contract DA-36-039-sc-87450